# LECTURE NOTES

ON

# CRYSTALLOGRAPHY

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COMPLIMENTS OF

**Colorado State School of Mines,**

GOLDEN, COLORADO

PUBLISHED BY THE AUTHOR,
GOLDEN, COLORADO

Colorado State School
of Mines



NEWS PRINTING COMPANY,
1896.

## PREFACE.

The difficulty of presenting the subject of crystallography introductory to a course in mineralogy, with only a limited time at one's disposal, has been realized by all instructors in this branch. Even the best text books fail to present the subject in so clear a light as to do away with the necessity of lecturing. The main difficulty with the best treatises on this subject, when put in the hands of the student, is their length and multiplicity of details.

These lecture notes are not intended to form a treatise on crystallography. They were originally prepared, not for publication, but for use in the class room, supplemented by lectures, crystal models and natural crystals. In the very

### ERRATA.

PAGE 8. Reverse the two signs >, <, in the table of forms.

PAGE 11. Between the second and third lines of italics at the top insert—all the faces that lie wholly within.

PAGE 11. Center of page. Should read—Trisoctahedron- - - -gives Tetragonal tristetrahedron.

etc. These omissions are made for the reason that such features are usually clearly presented in almost any mineralogy.

On account of the brevity of these notes the interleaved blank pages have been found very useful, and they will undoubtedly prove serviceable in case any one else should wish to use these notes as a basis for instruction.

STATE SCHOOL OF MINES, GOLDEN, COLO., April, 1896.

# CRYSTALLOGRAPHY.

In the inorganic world bodies are met with that are homogeneous throughout; others, again, that consist of two or more kinds of homogeneous substances; and still others that are composed of many distinct grains or particles of one kind of substance. The first are termed *Minerals*, the last two *Rocks*.

*A Mineral*, therefore, may be defined as *an inorganic, natural, homogeneous body.*

A mineral represents a definite chemical compound, and its formation is controlled by the same laws that control the formation of all chemical compounds.

These natural laws of growth express themselves in many ways, e. g., through outward form and through various physical and optical properties of the mineral. These different manifestations, however, vary with and thus characterize the chemical combination.

As an illustration of these properties we may take what is called cleavage. Just as a block of wood cleaves in certain directions, dependent on the grain of the wood, i. e., on the arrangement of the wood cells, so, many minerals are said to cleave in certain definite directions. And this mineral cleavage is likewise due to a sort of grain in the mineral produced by the arrangement of the molecules. To illustrate, rock-salt cleaves always in three directions at right angles to each other, while crystallized carbonate of lime cleaves in three directions that make an angle of about 105° to each other.

Another, and more important, outward manifestation of the laws of mineral growth is seen in the natural external form. There are three ways in which minerals are usually formed, namely, through solution, through fusion and through sublimation. But in any case, unless interfered with by some external agency, the mineral particles in forming are usually found to be bounded on all sides by plane surfaces and to

EDGES. *The line of intersection between two faces is called an edge.*

ANGLES. *These are of two sorts. An interfacial angle is the plane angle formed by the intersection of two faces. A crystal angle is the solid angle formed by the intersection of three or more faces.*

SIMILAR EDGES AND ANGLES *are those formed by the intersection of the same number of planes, similarly placed.*

Upon bringing together all known crystal forms it is seen that they may all be grouped into six crystal systems that differ from each other by the number and kind of the symmetry planes present. Below will be found grouped in accordance with the presence of principal and common symmetry planes the

## SIX CRYSTAL SYSTEMS.

| | |
|---|---|
| **I. WITH 3 PRINCIPAL SYMMETRY PLANES.** | |
| (1) with six common symmetry planes. | **Isometric System.** |
| **II. WITH 1 PRINCIPAL SYMMETRY PLANE.** | |
| (2) a, with four common symmetry planes arranged in two pairs of perpendicular planes. | **Tetragonal System.** |
| (3) b, with six common symmetry planes arranged in two groups of three each, making angles of 60° to each other. | **Hexagonal System.** |
| **III. WITH NO PRINCIPAL SYMMETRY PLANE.** | |
| (4) a, with three common symmetry planes making right angles with each other. | **Orthorhombic System.** |
| (5) b, with one common symmetry plane. | **Monoclinic System.** |
| (6) c, with no symmetry plane whatever. | **Triclinic System.** |

**USE OF CRYSTAL AXES.**

In addition to the symmetrical arrangement of the faces it has been noticed that, however numerous may be the faces on a crystal, they bear further definite relationships to each other in that, within certain limits, the facial angles appear to be fixed. The determination of these facial angles

is an important part of the study of crystallography, and for this purpose it has been found desirable to refer all the faces of a crystal to three (sometimes four) arbitrarily chosen fixed lines. If we can determine the inclinations which the different faces make to these fixed lines we can thereby determine their mutual inclinations to each other. These fixed lines are called *Crystal Axes.*

Theoretically these crystal axes might be chosen making any desired angles to each other, provided the angles be known, but for the sake of simplicity it is found desirable to choose them so that they may bear some definite relationship to the symmetry of the crystal.

Therefore, we have the following

**GENERAL RULE FOR CHOOSING CRYSTAL AXES.**

In all the six systems the crystal axes are chosen to coincide with symmetry axes, as far as this is possible; a principal symmetry axis always being given the preference.

**PARAMETERS.**

All the crystal axes are supposed to meet and cross each other at the geometric centre of the crystal. In ascertaining the inclinations of the faces to each other by means of the crystal axes we conceive both faces and axes to be extended indefinitely, or until the plane of any face under consideration cuts all three axes, or until it is evident that the plane never will cut the axis. The relative distances at which the plane of a face cuts the three axes, measured from the centre of the axial cross, are called the *parameters* of the face. Experience has shown that the value of a parameter is either infinity or some small rational quantity. The parameter value infinity is expressed thus, $\infty$, and indicates that the face is parallel to the axis.

**CRYSTAL FORM.**

In studying crystals in any one system it will be noticed that, if we start with any face with known parameter values, in order that the law of symmetry of the system may

be fulfilled there must also occur a definite number of other faces with exactly the same parameter values as has the face we start with. In speaking of crystal form, then, we use the term in a technical sense and mean all the faces taken together that are required to complete the symmetry of the system. *Crystal form*, therefore, may be defined as *the sum of all those crystal faces, which the symmetry of a crystal demands, when one of them is present.*

**LAW OF SYMMETRY.**

*Both ends of either a symmetry or a crystal axis, and the ends of all interchangeable axes must be cut by the same number of crystal faces, similarly placed.*

This is a fundamentally important law, and holds good with certain modifications to be noted beyond.

## ISOMETRIC SYSTEM.

This system has three principal symmetry planes at right angles to each other; also six common symmetry planes lying intermediate between and diagonal to the principal symmetry planes, and making with each other angles of 60°, 90° and 120°.

The three principal symmetry planes divide the crystal body into eight solid parts called octants.

The three principal symmetry axes are interchangeable and, in accordance with the rule, are chosen as the three crystal axes. The law of symmetry, therefore, in this system demands that the six ends of the three interchangeable axes should be cut by the same number of planes, similarly placed, *i. e.*, the six ends of the crystal axes must come out in similarly located points. A little thought will also show that all eight octants must contain the same number of crystal faces.

Crystals in this system show an equal development in three directions.

## CRYSTAL FORMS IN THE ISOMETRIC SYSTEM.

In determining crystal forms it will suffice if we can determine for a form how any one of its faces, if extended, would cut the three crystal axes, inasmuch as all the other faces of the same form must cut the axes at the same distances. We can indicate how the plane of a face cuts the three axes by means of the three parameter values expressed in the form of a ratio. Thus, a : nb : mc, where the letters a, b and c represent the three crystal axes, and the coefficients m and n give the parameter values. In the isometric system, as all three axes are interchangeable, they must all be treated alike, and it is not necessary to distinguish between a, b and c. We will designate all three, therefore, by the same letter, a. The parameter values expressed in the form of a ratio may be called the SYMBOL of a crystal face, and also of the form to which the face belongs.

In the table given below will be found all the possible variations of crystal symbols in the isometric system, so arranged as to make it quite evident that no variations are left out.

ISOMETRIC HOLOHEDRAL FORMS.

| | | | | |
|---|---|---|---|---|
| Three axes cut alike | | a : a : a | Octahedron | 8 Faces |
| Two axes cut alike | Two axes cut at distance > the other | a : a : ma<br>a : a : ∞a | Trisoctahedron<br>Dodecahedron | 24 Faces<br>12 Faces |
| | Two axes cut at distance < the other | a : ma : ma<br>a : ∞a : ∞a | Trapezohedron<br>Hexahedron | 24 Faces<br>6 Faces |
| Three axes cut unlike | | a : ma : na<br>a : ma : ∞a | Hexoctahedron<br>Tetrahexahedron | 48 Faces<br>24 Faces |

Of these seven possible isometric forms, one has six faces, one eight, one twelve, three twenty-four, and one forty-eight. The hexahedron with six faces has its faces parallel to the principal symmetry planes. The dodecahedron with twelve faces has its faces parallel to the six common symmetry planes.

The twenty-four sided forms are the most confusing. One of them may be readily determined by noting that its faces are each parallel to a crystal axis. In studying these, as in all other forms, first find the three principal symmetry axes, which are chosen as crystal axes. Place the crystal so that one axis is vertical and two horizontal, one of which runs from front to back and the other, necessarily, from right to left. For the two other twenty-four-sided forms, namely, the trapezohedron and the trisoctahedron, the following rule is very useful, provided that it is a question of deciding between these two forms only: The trisoctahedron has its faces so arranged in each octant that an *edge* runs from the centre of the octant out to each axis, while the trapezohedron has a *face* running from the centre of the octant out to each axis. (This rule will also apply in case of combined forms mentioned below.)

**COMBINATION OF FORMS.**

A crystal in this system may have any one of these seven forms, or it may have two or more, or even all seven developed at the same time. These forms, of course, must then mutually modify each other, and when so occuring cannot be expected to have the same shape as in the unmodified forms. On the other hand, the inclination of the faces to the axes cannot be in the least changed by such modification, so that each form may be determined by its symbol as readily as in the uncombined forms.

**RULE FOR COMBINATION OF FORMS.**

*First. All the faces of the same crystal form on the same crystal have the same shape and size.*

*Second. There are as many different crystal forms on a crystal as there are different kinds of faces.*

(The above rules hold only for crystal models or for perfectly developed natural crystals).

The following terms are used in describing combined forms:

*A face is said to replace an edge when it cuts the crystal parallel to the replaced edge.*

*An edge or angle is truncated when it is replaced by a face making equal angles with the two adjacent faces.*

*An edge is bevelled when it is replaced by two faces that are equally inclined to the faces making the replaced edge.*

### HOLOHEDRAL, HEMIHEDRAL AND TETARTOHEDRAL FORMS.

Each of the above described seven forms have all the faces that the fully developed symmetry of the system demands, hence they are called *Holohedral Forms.* This term is used to distinguish them from other forms that have only half of the faces developed that the symmetry of the system demands. Such forms are designated *Hemihedral Forms.* Still, others having but one-fourth of the faces developed are called *Tetartohedral Forms.*

There are many ways in which half the faces of a crystal form might be selected, and nature actually does choose more than one way, but the faces are not selected without regard to rule, for in all cases there must hold true the following

### LAW OF SYMMETRY OF HEMIHEDRAL AND TETARTOHEDRAL FORMS.

*The ends of all similar (holohedral) symmetry axes must be cut by the same number of crystal faces, similarly placed.*

The following rule is important as to

NUMBER OF HEMIHEDRAL AND TETARTOHEDRAL FORMS. *Every holohedral form has a corresponding hemihedral or tetartohedral form.*

The above law and rule can only hold true when the parts to be developed are obtained by cutting the holohedral forms into sections by means of symmetry planes. The kind of hemihedrism will depend upon the choice of these symmetry planes. There are thus three possible hemihedrisms in the isometric system. The first obtained by cutting the holohedral forms by means of the principal symmetry planes; the second, by using for this purpose the common symmetry planes; the third, by using both sets of symmetry planes.

## I. INCLINED HEMIHEDRAL FORMS IN THE ISOMETRIC SYSTEM.

*Inclined hemihedral forms may be conceived to be developed by suppressing on each of the seven holohedral forms four alternating octants (obtained by dividing the holohedral form by the three principal symmetry planes), while the remaining faces are developed.*

By this process we suppress faces on one side of a principal symmetry plane and develop on the other side. This necessarily destroys the principal symmetry plane.

*Inclined hemihedral forms, therefore, may be distinguished by the fact that they have the six common symmetry planes, but not the principal symmetry planes of the isometric system.*

The following forms result from the application of the above law.

| | | | |
|---|---|---|---|
| Octahedron | gives | Tetrahedron | with 4 faces |
| Trapezohedron | gives | Trigonal tristetrahedron | with 12 faces |
| Trisoctahedron | gives | Tetragonal trisoctahedron | with 12 faces |
| Hexoctahedron | gives | Hextetrahedron | with 24 faces |
| Hexahedron | gives | Hexahedron | with 6 faces |
| Dodecahedron | gives | Dodecahedron | with 12 faces |
| Tetrahexahedron | gives | Tetrahexahedron | with 24 faces |

The first four of the above forms occur with just half as many faces as have the corresponding holohedral forms, but the last three occur with all the faces present in the holohedral forms. They may appear to be holohedral, but are really hemihedral forms. Hemihedral forms, therefore, cannot always be distinguished from holohedral by the number of their faces. The reason why these three forms occur with all the faces of the corresponding holohedral forms is evident when we consider that they have infinity in their symbol, *i. e.*, each face, being parallel to an axis, must lie in two adjacent octants and cannot, therefore, be suppressed according to the rule for inclined hemihedral forms.

## II. PARALLEL HEMIHEDRAL FORMS IN THE ISOMETRIC SYSTEM.

*Parallel hemihedral forms may be conceived to be developed by suppressing on each of the seven holohedral forms all*

*the faces that lie wholly within twelve alternating parts obtained by dividing the holohedral form by means of the common symmetry planes, while the remaining faces are developed.*

By this process we destroy planes on one side of a common symmetry plane and develop those on the other side. This necessarily destroys the common symmetry planes.

*Parallel hemihedral forms, therefore, may be distinguised by the fact that they have the three principal symmetry planes but not the six common symmetry planes of the isometric system.*

The following forms result from the application of the above law :

| | | |
|---|---|---|
| Tetrahexahedron ....gives | Pentagonal dodecahedron..with | 12 faces |
| Hexoctahedron ......gives | Didodecahedron ..........with | 24 faces |
| Octahedron .........gives | Octahedron ..............with | 8 faces |
| Hexahedron .........gives | Hexahedron...............with | 6 faces |
| Dodecahedron........gives | Dodecahedron ............with | 12 faces |
| Trapezohedron ......gives | Trapezohedron ...........with | 24 faces |
| Trisoctahedron .....gives | Trisoctahedron ..........with | 24 faces |

In this case there are only two forms that occur with half as many faces as do the corresponding holohedral forms. The last five forms, though apparently holohedral, are really hemihedral.

The reason why five forms occur with as many faces in the parallel hemihedral division as in holohedral is similar to that given for the inclined hemihedral forms. Each face is so placed that it lies in at least two adjacent parts, and therefore cannot be suppressed.

### III. GYROIDAL HEMIHEDRAL FORMS IN THE ISOMETRIC SYSTEM

*Gyroidal hemihedral forms may be conceived to be developed by suppressing on each of the seven holohedral forms the faces lying wholly within twenty-four alternating parts obtained by cutting the holohedral forms into forty-eight sections by means of both sets of symmetry planes, while the remaining faces are developed.*

By this process all the symmetry planes are destroyed, and gyroidal hemihedral forms may be distinguished by this fact.

As there is but one holohedral form with forty-eight faces, this can evidently be the only form whose faces lie wholly within the forty-eight parts into which the nine symmetry planes cut a crystal. Therefore, the hexoctahedron can be the only form giving a new hemihedral form. This is called the *pentagonal icositetrahedron.* There are two of these, distinguished as right and left-handed, differing from each other only as a right-handed glove differs from a left-handed.

Gyroidal hemihedral forms are very rare and are of no importance from a practical point of view.

**TETARTOHEDRAL FORMS IN THE ISOMETRIC SYSTEM.**

Tetartohedral forms are also very rare in the isometric system. As a description of these forms is not thought to be in accord with the object of these notes, the reader is referred to larger treatises on crystallography and mineralogy. The principles upon which tetartohedral forms are conceived to be developed will be found set forth in these Lecture Notes under the hexagonal system.

**GENERAL REMARKS ON HEMIHEDRAL AND TETARTOHEDRAL FORMS.**

No substance crystallizes both holohedral and hemihedral or tetartohedral, and two kinds of hemihedral forms, or hemihedral and tetartohedral forms are never found on the same crystal. We do find, however, *apparantly holohedral,* (but *really hemihedral* forms) occurring with other hemihedral forms.

The symbols of hemihedral forms are the same as those of holohedral forms, but they are written in the form of a fraction with 2 for a denominator. Similarly, tetartohedral forms have the denominator 4.

Holohedral forms really give two hemihedral forms which are usually exactly alike, except in position, and are designated as positive and negative. Thus, the octahedron gives a + tetrahedron and a — tetrahedron. The same holds true for all the systems.

TABLE GIVING THE SYMBOLS OF THE FORMS IN THE ISOMETRIC SYSTEM AS USED BY WEISS, NAUMANN, DANA AND MILLER.

| | WEISS. | NAUMANN. | DANA. | MILLER. |
|---|---|---|---|---|
| Octahedron | a : a : a | O | 1 | (111) |
| Hexahedron | a : ∞a : ∞a | ∞O∞ | H | (100) |
| Dodecahedron | a : a : ∞a | ∞O | i | (110) |
| Trisoctahedron | a : a : ma | mO | m | (hhl) |
| Trapezohedron | a : ma : ma | mOm | m-m | (hll) |
| Tetrahexahedron | a : ma : ∞a | ∞On | i-n | (hk0) |
| Hexoctahedron | a : ma : na | mOn | m-n | (hkl) |

## TETRAGONAL SYSTEM.

In this system there is but one principal symmetry plane and four common symmetry planes all at right angles to the principal symmetry plane. These four common symmetry planes occur as two pairs of right angled planes, each pair standing intermediate between or 45° inclined to the other pair.

There is, therefore, one direction, namely, that of the principal symmetry axis independent of and distinguished from the others. The two common symmetry axes of each pair are interchangeable with each other, but not with the principal symmetry axis, nor with the common symmetry axes of the other pair.

*In choosing the three crystal axes we will, in accordance with the general rule, select the principal symmetry axis for one crystal axis, and for the two other crystal axes we will select either one of the two pairs of common symmetry axes.*

In studying the crystal forms it is customary to place the principal symmetry plane horizontal. The principal symmetry axis, therefore, becomes the vertical crystal axis. The two other selected crystal axes are horizontal, one of them being placed from front to back. The two horizontal axes, being interchangeable, are both designated by the letter a, and the vertical axis by the letter c.

*The vertical axis c can never be equal to the horizontal axis a, and the ratio between c and a can never be a rational*

*quantity.* In the symbols given below, however, the coefficients before c and a are always rational quantities and may also be equal.

As 1c can never be equal to 1a, m may become equal to 1 without changing the character of the form.

HOLOHEDRAL TETRAGONAL FORMS.

| | | | |
|---|---|---|---|
| First Order | a : a : mc<br>a : a : ∞c | Direct pyramid<br>Direct prism | 8 Faces<br>4 Faces |
| | a : na : mc<br>a : na : ∞c | Ditetragonal pyramid<br>Ditetragonal prism | 16 Faces<br>8 Faces |
| Second Order | a : ∞a : mc<br>a : ∞a : ∞c | Indirect pyramid<br>Indirect prism | 8 Faces<br>4 Faces |
| | ∞a : ∞a : c | Basal pinacoid | 2 Faces |

COMBINATION OF FORMS. Only in case of one of the three pyramids can a single crystal form entirely bound a crystal. The other forms can occur only in combination.

The basal pinacoid and the prisms of the first and second orders have no variable in their symbol. They can occur therefore but once on a crystal. All the other forms are variable and can occur an indefinite number of times on the same crystal.

The forms in this system can be readily determined without the aid of the symbols by means of the following

**RULES FOR DETERMINING TETRAGONAL FORMS.**

*First.* A face which is parallel to both horizontal axes is the basal pinacoid.

*Second.* A face whose plane cuts all three axes belongs to a pyramid. If it cuts the two horizontal axes alike it belongs to a direct pyramid; if unlike, to a ditetragonal pyramid.

*Third.* A face whose plane cuts the vertical axis and is parallel to one of the horizontal axes belongs to an indirect pyramid.

*Fourth.* A face parallel to the vertical axis belongs to a prism. If the plane of the face cuts both horizontal axes alike it belongs to a direct prism; if unlike, to a ditetragonal prism. If the plane cuts one horizontal axis and is parallel to the other, it belongs to an indirect prism.

*Fifth.* If two or more pyramids, or a prism and pyramid, make with each other horizontal edges, they are of the same order.

**HEMIHEDRAL FORMS IN THE TETRAGONAL SYSTEM.**

There are three possible kinds of hemihedrism in this system, and these are closely analogous with those in the isometric system in their method of development.

I. SPHENOIDAL HEMIHEDRISM. As is the case with inclined hemihedral forms, these are developed by dividing the holohedral forms by means of the principal symmetry plane, and by the two symmetry planes containing the horizontal axes. These give us octants, which are alternately suppressed and developed.

From the pyramid of the first order there is obtained the *sphenoid*. This is equivalent to the tetrahedron, from which it differs in that the two horizontal edges are different from the four other edges.

From the ditetragonal pyramid there is developed the *tetragonal scalenohedron*, which does not closely resemble any isometric form.

These two are the only new forms, all the others being externally identical with the corresponding holohedral forms.

**II. PYRAMIDAL HEMIHEDRISM.**

To develop these forms we conceive each of the holohedral forms to be cut into eight parts by means of both sets of common symmetry axes.

The ditetragonal pyramid gives us a new form called the *pyramid of the third order*, which differs in no respect

from the pyramids of the first and second orders, except that it occupies a position unsymmetrical with reference to the common symmetry planes and to the other forms.

Similarly the ditetragonal prism gives the *prism of the third order* with similar relationships to the symmetry planes and to the other forms.

All the other forms occur as though they were holohedral.

### III. TRAPEZOHEDRAL HEMIHEDRISM.

To develop these forms we conceive each of the holohedral forms to be cut by means of all five symmetry planes into sixteen parts, each corresponding in position to that of the ditetragonal pyramid.

The ditetragonal pryamid, obviously, is the only form that can give a new form. It is called the *tetragonal trapezohedron.*

There are no known natural minerals crystallizing in this form, although certain organic salts are known to belong to this division.

### TETARTOHEDRAL FORMS IN THE TETRAGONAL SYSTEM.

As no minerals are absolutely known to crystallize tetartohedral in this system, the possible tetartohedral divisions are omitted. They are exactly analogous to those that will be found described under the hexagonal system.

TABLE OF SYMBOLS IN THE TETRAGONAL SYSTEM.

| | WEISS | NAUMANN | DANA | MILLER |
|---|---|---|---|---|
| Direct pyramid | a : a : mc | mP | m | (hhl) |
| Indirect pyramid | a : ∞a : mc | mP∞ | m-i | (h0l) |
| Ditetragonal pyramid | a : na : mc | mPn | m-n | (hkl) |
| Direct prism | a : a : ∞c | ∞P | I | (110) |
| Indirect prism | a : ∞a : ∞c | ∞P∞ | i-i | (100) |
| Ditetragonal prism | a : na : ∞c | ∞Pn | i-n | (hk0) |
| Basal pinacoid | ∞a : ∞a : mc | 0P | O | (001) |

**COMBINATION OF FORMS.**

As is the case in the tetragonal system the pyramids are the only forms that can wholly bound a crystal. The other forms can occur only in combination. The basal pinacoid and the prisms of the first and second order, having no variable in their symbol, can occur but once, while the other forms with variable symbols can occur an indefinite number of times on the same crystal.

The forms in this system may be readily determined without the aid of symbols by means of the following

**RULES FOR DETERMINING HEXAGONAL FORMS.**

*First.* A face which is parallel to the horizontal axes is the basal pinacoid.

*Second.* A face whose plane cuts the vertical axis obliquely belongs to a pyramid. If the plane cuts two of the lateral axes equally and is parallel to the third the pyramid is of the *first order*. If one horizontal axis is cut at unity and the two others at twice the distance the pyramid is of the *second order*. If the three horizontal axes are all cut unequally the pyramid is *dihexagonal.*

*Third.* A face parallel to the vertical axis belongs to a prism. If the plane cuts two horizontal axes equally and is parallel to the third the prism is of the *first order*. If one horizontal axis is cut at unity (perpendicularly) and the two others at twice the distance the prism is of the *second order.* If the three horizontal axes are all cut unequally the prism is *dihexagonal.*

*Fourth.* If two or more pyramids or a prism and a pyramid make with each other horizontal edges they are of the same order.

**HEMIHEDRAL FORMS IN THE HEXAGONAL SYSTEM.**

The hemihedral forms in the hexagonal system are of greater importance than those of any other system, with the possible exception of the isometric system, and are, therefore, treated at greater length.

There are three possible kinds of hemihedrism in this system, corresponding to those given for the tetragonal system.

**I. RHOMBOHEDRAL HEMIHEDRISM.**

This kind of hemihedrism may be conceived to be developed by cutting each of the seven holohedral forms into twelve parts (called dodecants) by means of the principal symmetry plane and by one set of common symmetry planes, namely, by the three planes in which the horizontal axes have been chosen; and by suppressing all the faces that lie wholly within six alternating dodecants, while all the remaining faces are developed.

In case of five of the holohedral forms, namely, basal pinacoid, first and second order prisms, second order pyramid and dihexagonal prism, no one face lies wholly within one dodecant, therefore, no face can be suppressed in accordance with the above stated law. These five forms, therefore, have hemihedral forms exactly like the corresponding holohedral forms, and as such they may occur in combination with the apparently as well as really hemihedral forms given below.

Two of the holohedral forms give new half forms. The *dihexagonal pyramid* gives the *scalenohedron*. The *pyramid of the first order* gives the *rhombohedron*.

The scalenohedron is a twelve sided form that may be distinguished from the hexagonal pyramid by the following facts: First, the lateral edges are not horizontal, but zigzagged. Second, the edges running to the vertex are alternately sharper and blunter. The sharper edge above lies over the blunter edge below and vice versa.

The rhombohedron is a six sided form, with three faces above and three below. The following properties may be noted: First, the three edges running to the vertex are equal, but are different from the zig-zag lateral edges. Second, the upper faces do not lie over the under faces, but alternate with them.

In general, rhombohedral hemihedral forms may be recognized by the fact that they have only three symmetry planes, namely, the three planes lying intermediate between the crystal axes.

## II. PYRAMIDAL HEMIHEDRISM.

This kind of hemihedrism may be conceived to be developed by cutting each of the seven holohedral forms into twelve parts, or dodecants, by means of the two sets of common symmetry planes; and by suppressing all the faces that lie wholly within six alternating dodecants, while the remaining faces are developed.

Here, too, we find that five of the holohedral forms cannot give new half forms inasmuch as they do not have their faces lying wholly within the dodecants. These forms are basal pinacoid, pyramid and prism of the first order and pyramid and prism of the second order. These five forms, therefore, if they occur at all, must occur with all their faces.

The two other forms are:

*Dihexagonal pyramid*, gives *pyramid of the third order*, with twelve faces.

*Dihexagonal prism*, gives *prism of the third order*, with six faces.

The third order pyramid and prism, differ in no respect from the first or second order pyramid and prism except in position. They lie unsymmetrical with reference to the horizontal axes, *i. e.*, they cut the horizontal axes unequally. When they occur in combination with other prisms and pyramids, their unsymmetrical position is easily noted.

In general, the pyrimidal hemihedral forms may be recognized by the fact that they have only the principal symmetry plane present.

## III. TRAPEZOHEDRAL HEMIHEDRISM.

These forms may be conceived to be developed by cutting each of the seven holohedral forms into twenty-four parts by means of all seven symmetry planes; and by suppressing all the faces that lie wholly within twelve alternating parts, while all the remaining faces are developed.

In this case there is only one form whose faces lie wholly within these twenty-four parts, namely, the dihexagonal pyramid. Therefore, this is the only form that can give a

new hemihedral form. This form is called the hexagonal trapezohedron. It consists of six faces above, that lie neither exactly above nor exactly alternating with the six below.

In general, this form may be recognized by the absence of all symmetry planes, combined with an hexagonal arrangement of the faces.

**TETARTOHEDRAL FORMS IN THE HEXAGONAL SYSTEM.**

Tetartohedral or quarter forms may be conceived to be developed from the holohedral forms by the simultaneous or successive application to each of these forms of two kinds of hemihedrism. As there are three kinds of hemihedrism, there are three possible combinations to be considered. *In every tetartohedral form, however, the following must hold true: The opposite ends of a (holohedral) symmetry axis and ends of all interchangeable (holohedral) symmetry axes must be cut by the same number of planes similarly placed.*

We will now see whether the above condition can be fulfilled by combining, first, the rhombohedral and trapezohedral hemihedrisms; second, the rhombohedral and pyramidal hemihedrisms; third, the pyramidal and trapezohedral hemihedrisms.

If a tetartohedral form is possible it can certainly be developed on the most general form, the dihexagonal pyramid. We will apply the test to this form for the three cases. The figures below represent the twenty-four faces of the dihexagonal pyramid arranged in an upper and a lower bank to correspond with the position of the faces on the holohedral form. (This method is taken from Prof. Groth's well known work on Physical Crystallography.)*

The faces that would be suppressed by the application of the rhombohedral hemihedrism alone are cancelled by diagonal lines, thus, /; those that would be suppressed by the application of the trapezohedral hemihedrism are cancelled by diagonal lines, thus, \; and those that would

* P. Groth. Physikalische Krystallographie. Leipzig. Wilhelm Engelmann.

be suppressed through the pyramidal hemihedrism are cancelled by horizontal lines.

*First. Suppression through rhombohedral and trapezohedral hemihedrisms.*

Upper faces ~~1~~ 2 ~~3~~ ~~4~~ ~~5~~ 6 ~~7~~ ~~8~~ ~~9~~ 10 ~~11~~ ~~12~~
Lower faces ~~1~~ ~~2~~ 3 ~~4~~ ~~5~~ ~~6~~ 7 ~~8~~ ~~9~~ ~~10~~ 11 ~~12~~

Leaving faces 2, 6, 10 above, and 3, 7, 11 below to be developed.

*Second. Suppression through rhombohedral and pyramidal hemihedrisms.*

Upper faces ~~1~~ 2 ~~3~~ ~~4~~ ~~5~~ 6 ~~7~~ ~~8~~ ~~9~~ 10 ~~11~~ ~~12~~
Lower faces ~~1~~ ~~2~~ ~~3~~ 4 ~~5~~ ~~6~~ ~~7~~ 8 ~~9~~ ~~10~~ ~~11~~ 12

Leaving faces 2, 6, 10 above, and 4, 8, 12 below to be developed.

*Third. Suppression through pyramidal and trapezohedral hemihedrisms.*

Upper faces ~~1~~ 2 ~~3~~ 4 ~~5~~ 6 ~~7~~ 8 ~~9~~ 10 ~~11~~ 12
Lower faces ~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ ~~6~~ ~~7~~ ~~8~~ ~~9~~ ~~10~~ ~~11~~ ~~12~~

Leaving faces 2, 4, 6, 8, 10, 12 above, and none below.

It is evident that the last case does not give a form that fulfills the above conditions, but these conditions are found to be fulfilled in the first two cases. We have, therefore, two possible kinds of tetartohedrism. First. The *trapezohedral tetartohedrism*, developed by the simultaneous application of the rhombohedral and the trapezohedral hemihedrisms, and second, the *rhombohedral tetartohedrism*, developed by the simultaneous application of the rhombohedral and the pyramidal hemihedrisms.

### I. TRAPEZOHEDRAL TETARTOHEDRISM.

If this method be applied to the seven holohedral forms it is found that two of them can give no new form, as none of their faces can be suppressed. They are the basal pinacoid and the prism of the first order.

The prism of the third order differs from the prisms of the first and second orders only in its position, it being unsymmetrical with reference to the crystal axes. Similarly, the third order rhombohedron has an unsymmetrical position, but otherwise does not differ from the other rhombohedrons.

The symbols of tetartohedral forms may be expressed in the form of a fraction with the usual holohedral symbols for numerators and 4 for the denominator.

For a further description of the symbols as well as for fuller descriptions of these tetartohedral forms the reader is referred to more extended treatises on crystallography.*

## HEMIMORPHIC HEXAGONAL FORMS.

Hemimorphic forms are half-forms that differ from hemihedral forms in a very important respect, namely, the opposite ends of some (holohedral) symmetry axis are cut by different planes. Usually some form or forms that are present on one end are wanting at the other end of the axis.

In the hexagonal system hemimorphic forms play a very important and interesting role in the case of a very common mineral, tourmaline. Here we have the hemimorphism superimposed upon a rhombohedral hemihedrism. This gives a sort of quarter-form. The forms that commonly occur hemimorphic in this mineral, *i. e.*, differently developed at the two ends of the vertical axis, are rhombohedrons, scalenohedrons and the basal pinacoid. In addition to these there are usually to be seen a trigonal prism of the first order, a ditrigonal prism and the prism of the second order.

These forms differ from tetartohedral forms in that we have a trigonal prism in combination with a scalenohedron, and by the fact that the trigonal prism is of the first order, instead of the second order. The trigonal prism, therefore,

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* See Elements of Crystallography, by George H. Williams, New York, Henry Holt & Co., 1890.

lies under the rhombohedron, *i. e.*, makes a horizontal edge with the rhombohedron.

The explanation of the trigonal prism is as follows: The hexagonal prism of the first order may be considered to be the equivalent of a rhombohedron with infinite c axis, three faces belonging to the upper and three to the lower half of the crystal. If, now, the crystal is hemimorphic, the three prism faces that belong to one end of the crystal may be developed while the three belonging to the other end may be suppressed. In the same way the ditrigonal prism may in this case be considered to be the hemimorphic form of a scalenohedron with infinite c axis.

TABLE OF SYMBOLS IN THE HEXAGONAL SYSTEM.

| | WEISS. | NAUMANN. | DANA. | MILLER-BRAVAIS. |
|---|---|---|---|---|
| First order pyramid. | a : a : ∞a : mc | mP | m | $(h0\bar{h}i)$ |
| Second order pyramid | 2a : a : 2a : mc | mP2 | m-2 | $(hh\bar{2h}2i)$ |
| Dihexagonal pyramid | pa : a : na : mc | mPn | m-n | $(hk\bar{l}i)$ |
| First order prism.... | a : a : ∞a : ∞c | ∞P | I | $(10\bar{1}0)$ |
| Second order prism.. | 2a : a : 2a : ∞c | ∞P2 | i-2 | $(10\bar{2}0)$ |
| Dihexagonal prism.. | pa : a : na : ∞c | ∞Pn | i-n | $(kh\bar{l}o)$ |
| Basal pinacoid...... | ∞a : ∞a : ∞a : c | 0P | O | (0001) |

## ORTHORHOMBIC SYSTEM.

This system belongs to the class with no principal symmetry plane. It has, however, three common symmetry planes at right angles to each other. There are, therefore, no interchangeable symmetry axes.

In accordance with the rule for the selection of crystal axes the three common symmetry axes become the crystal axes. None of these axes is pre-eminent above the others, as there is no principal symmetry axis. We may select, then, any one of the three axes for the vertical axis, the two others becoming the horizontal axes. The shorter of the horizontal axes is placed from front to back and is called

the brachy (short) axis. It is designated by the letter a. The longer horizontal axis runs, therefore, from right to left, and is called the macro (long) axis. It is designated by the letter b. The vertical axis is designated by the letter c.

In this and the following systems the unit values for the three axes are different, and the ratio between them is an irrational one. On the other hand the co-efficient values are always simple rational quantities.

**CRYSTAL FORMS IN THE ORTHORHOMBIC SYSTEM.**

There are three kinds of forms in this system.

*First.* Forms with eight faces..............Pyramids.
*Second.* Forms with four faces............Prism and Domes.
*Third.* Forms with two faces.............Pinacoids.

| | | | | | |
|---|---|---|---|---|---|
| Pyramid | na | : | b | : | mc |
| Prism | na | : | b | : | ∞c |
| Macro-dome | a | : | ∞b | : | mc |
| Brachy-dome | ∞a | : | b | : | mc |
| Macro-pinacoid | a | : | ∞b | : | ∞c |
| Brachy-pinacoid | ∞a | : | b | : | ∞c |
| Basal pinacoid | ∞a | : | ∞b | : | c |

Pyramids cut all three axes. Prisms and domes cut two axes and are parallel to one. Pinacoids cut but one and are parallel to two axes.

*The relative lengths of any two axes cannot be determined by the thickness of a crystal, but only by means of some face which, if extended, would cut the two axes in question. E.g.*, the prism face which cuts the two horizontal axes determines which is the longer and which is the shorter. The dome determines the relative lengths of the vertical and one of the horizontal axes. The pyramid face, which cuts all three axes determines the relative lengths of the three axes.

It is evident that the prism and the domes are virtually the same thing with different names, as we can change either dome into a prism by a different selection of the vertical axis. In a certain sense, therefore, the domes and the prism are interchangeable. In the same sense the three pinacoids are also interchangeable.

## RULES FOR DETERMINING FORMS IN THE ORTHORHOMBIC SYSTEM.

*First.* A face whose plane cuts all three axes belongs to a pyramid.

*Second.* A face parallel to one axis belongs either to a prism or to a dome; if parallel to the vertical axis it belongs to a prism; if parallel to the brachy (short) axis, to a brachy-dome; if parallel to the macro (long) axis, to a macro-dome.

*Third.* A face parallel to two axes belongs to a pinacoid; if parallel to the vertical and brachy axes it belongs to the brachy-pinacoid; if parallel to the vertical and macro axes, to the macro-pinacoid; if parallel to the two horizontal axes, to the basal pinacoid.

*Fourth.* There may occur on the same crystal an indefinite number of pyramids or of prisms or domes, because these forms have variable symbols; but the three pinacoids can occur but once, as their symbols are invariable.

## HEMIHEDRAL AND HEMIMORPHIC FORMS IN THE ORTHORHOMBIC SYSTEM.

There is but one kind of hemihedrism possible in this system, namely, the one corresponding to the inclined in the isometric system, and to the sphenoidal in the tetragonal system. This is produced by cutting the holohedral forms by means of the common symmetry planes into octants, and by suppressing the faces that lie wholly within alternating octants while the remaining faces are developed.

There is only one form that can produce a new form, and this is the pyramid. This produces the *orthorhombic sphenoid*, which is similar to the tetragonal sphenoid, but without interchangeable axes.

*Hemimorphic forms* are fairly common in this system. They are developed by suppressing certain forms at one end of a symmetry axis that are developed at the other end.

TABLE OF SYMBOLS IN THE ORTHORHOMBIC SYSTEM.

| | WEISS. | NAUMANN. | DANA. | MILLER. |
|---|---|---|---|---|
| Pyramid | na : b : mc | mPn | m-n | (hkl) |
| Prism | na : b : ∞c | mP | I | (110) |
| Brachy-dome | ∞a : b : mc | $\infty P\breve{n}$ | $i\text{-}\breve{n}$ | (kh0) h > k |
| Macro-dome | a : ∞b : mc | $\infty P\bar{n}$ | $i\text{-}\bar{n}$ | (hk0) h > k |
| Brachy-pinacoid | ∞a : b : ∞c | $\infty P\breve{\infty}$ | $i\text{-}\breve{i}$ | (010) |
| Macro-pinacoid | a : ∞b : ∞c | $\infty P\bar{\infty}$ | $i\text{-}\bar{i}$ | (100) |
| Basal pinacoid | ∞a : ∞b : c | 0P | O | (001) |

## MONOCLINIC SYSTEM.

In this system there is but one symmetry plane and, therefore, but one symmetry axis.

In accordance with the rule we select this symmetry axis for one of the crystal axes. It is the only natural crystal axis.

The two remaining axes are selected to lie in the symmetry plane and parallel to prominent edges, or, if no edge is available, parallel to a prominent face. (Only in rare exceptions must a line connecting two corners be chosen). If an axis be parallel to an edge it must necessarily be parallel to the faces forming the edge.

Now, as edges parallel to the symmetry plane in the monoclinic system are not found to occur exactly at right angles to each other, the two axes in the symmetry plane are also never at right angles.

*We have, then, two axes lying in the symmetry plane oblique to each other, and one axis coinciding with the symmetry axis, and, therefore, at right angles to the other two axes.*

In orienting the crystal the symmetry plane is placed vertical and from front to back, so that the symmetry axis becomes the b axis, and is called the *ortho-axis.* The crystal is then turned until one of the oblique axes becomes vertical and the other slopes downward from the center of

the crystal towards the observer, *i. e.*, towards the front. The vertical axis then becomes the c axis; and the other, or *clino-axis* the a axis.

### CRYSTAL FORMS IN THE MONOCLINIC SYSTEM.

*First.* Forms with four faces—prisms, clino-domes and pyramids.

*Second.* Forms with two faces—ortho-domes and pinacoids.

| | | | | | |
|---|---|---|---|---|---|
| *First.* | Pyramids | Positive | $-na$ : | $b$ : | $mc$ |
| | | Negative | $+na$ : | $b$ : | $mc$ |
| | Prism | | $na$ : | $b$ : | $\infty c$ |
| | Clino-dome | | $\infty a$ : | $b$ : | $mc$ |
| *Second.* | Ortho-domes | Positive | $-a$ : | $\infty b$ : | $mc$ |
| | | Negative | $+a$ : | $\infty b$ : | $mc$ |
| | Ortho-pinacoid | | $a$ : | $\infty b$ : | $\infty c$ |
| | Basal pinacoid | | $\infty a$ : | $\infty b$ : | $c$ |
| | Clino-pinacoid | | $\infty a$ : | $b$ : | $\infty c$ |

The plane in which lie the vertical and the ortho-axes divides the crystal into two unsymmetrical parts. Therefore, the faces that may occur on one side of this plane are not repeated on the other side, except that every face must have an opposite parallel face. For instance, a face in the pyramid position cutting the vertical and ortho-axes and the front end of the clino axis need not occur at the rear of the crystal in a symmetrical position. The two pyramid faces in front, above the basal pinacoid, together with the two opposite and parallel faces at the back, below the basal pinacoid, make up a monoclinic pyramid which is called a partial pyramid. It takes two of these partial pyramids to correspond with the orthorhombic pyramid.

For a similar reason there are two partial ortho-domes. In the case of these partial forms if a face cuts the ends of the vertical and clino axes that form an obtuse angle it belongs to a negative partial form. If it cuts the ends of these axes forming the acute angle the form is positive.

The pyramid, prism and clino-dome are virtually the same thing, inasmuch as they can be changed the one into the other by changing the locations of the two oblique axes.

For the same reason the basal pinacoid, ortho-pinacoid and ortho-dome may be changed the one into the other.

The clino-pinacoid is the only form that cannot be changed into an other by changing the position of the oblique axes, for the reason that it is the only form parallel to a symmetry plane.

### RULES FOR DETERMINING FORMS IN THE MONOCLINIC SYSTEM.

*First.* A face whose plane cuts all three axes belongs to a pyramid. If it lies over the acute angle of the oblique axes it belongs to a positive pyramid; if over the abtuse angle, to a negative pyramid.

*Second.* A face parallel only the vertical axis belongs to a prism.

*Third.* A face parallel only the clino-axis belongs to a clino-dome.

*Fourth.* A face parallel only to the ortho-axis belongs to an ortho-dome.

*Fifth.* A face parallel to two axes is a pinacoid; if it is parallel to the clino and the ortho-axes it belongs to the basal pinacoid; if to the vertical and the clino-axes, to the clino-pinacoid; if parallel to the vertical and the ortho-axes, to the ortho-pinacoid.

*Sixth.* All the forms except the three pinacoids may occur an indefinite number of times on the same crystal because they have variable symbols.

### HEMIHEDRAL AND TETARTOHEDRAL FORMS IN THE MONOCLINIC SYSTEM.

Tetartohedral forms are not certainly known to occur. Hemihedral forms are known to exist, but their occurrence is so rare that a discussion of these forms is considered beyond the scope of these lecture notes.

them, as any form may be changed into any other form merely by making a different selection of the crystal axes.

Forms are named as they are in the orthorhombic system.

## RULES FOR DETERMINING CRYSTAL FORMS IN THE TRICLINIC SYSTEM.

*First.* All faces whose planes cut all three axes belong to pyramids. There are four of these partial pyramids to two in the monoclinic and to one in the orthorhombic systems.

*Second.* Faces parallel only to the vertical axis belong to prisms. There are two of these partial prisms.

*Third.* Faces parallel only to the brachy-axis belong to brachy-domes. There are two of these partial brachy-domes.

*Fourth.* Faces parallel only to the macro-axis belong to macro-domes. There are two of these partial macro-domes.

*Fifth.* Faces parallel to the brachy and macro-axes belong to the basal pinacoid.

*Sixth.* Faces parallel to the vertical and brachy-axes belong to the brachy-pinacoid.

*Seventh.* Faces parallel to the vertical and macro-axes belong to the macro-pinacoid.

*Eighth.* The pyramids, prisms, brachy and macro-domes, having variable symbols, may occur an indefinite number of times on a crystal ; all the other forms but once.

## SYMBOLS.

The symbols used are exactly the same as those given for the orthorhombic system. To distinguish between the different partial forms special accents and positive and negative signs are used.

## DISTORTION OF CRYSTALS.

In all the foregoing cases we have had under consideration ideally developed crystals or crystal models. In nature, however, perfect crystals are very rare. Almost invariably they are to some extent distorted. Except in the case of mechanical distortion this crystal distortion is usually of such a nature that the inclinations of the faces to each other and to the axes remain unaltered. This may be conceived to be accomplished by the shoving of one or more faces parallel to themselves so that some faces are inordinately developed at the expense of other faces. In this way some faces may be completely crowded off the crystal with an apparent loss of symmetry.

In natural crystals, therefore, we cannot recognize symmetry planes by the symmetrical development of faces on both sides of a plane, but merely by the equal inclinations of corresponding faces to the symmetry plane.